Real-Time Control of DSP Parametric Equalizers

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Network control brings new possibilities to user interaction with DSP devices. One important area is to offer real-time control of filter shapes (center frequency, Q, and gain) by changing filter coefficients in real time. Textbook solutions to DSP filter problems rarely consider such dynamic situations. This paper presents observations on implementing real-time control of DSP. The work explores the update rate and granularity required to achieve perceptually smooth, real-time control.

1 INTRODUCTION

The availability of high performance, low cost Digital Signal Processors (DSPs) has brought newfound flexibility to the design of professional audio equipment. This, coupled with advances in real-time network technology brings about new possibilities, offering new solutions to audio engineers. The advantages of audio processing in the digital domain are well known. High resolution Analog-to-Digital (A/D) and Digital-to-Analog (D/A) converters together with DSPs now make possible the transparent replacement of many traditional analog audio processing functions. DSP also makes possible very precise room and loudspeaker equalization, unheard of with analog circuitry. The flexibility of digital processing allows general purpose equipment to be produced and later configured in the field to meet the requirements of various applications.

Despite these benefits, DSP is not perfect. One major disadvantage of digital processing arises from the fact that it is a discrete approximation of a continuous analog function. In many cases, such as in studio recording or during a live performance, continuous control of filter parameters is required. In a digital implementation, errors, instabilities, and undesirable audible artifacts can occur when filter parameters are updated. In addition, the requirements for real-time update of filter parameters can result in significant computational burden, which increases the cost of the system.

Reference [1] provides a good theoretical discussion of the causes and conditions of audibility of distortions in implementing time-varying digital filters. This reference indicates that the audibility is a function of the input signal, the filter parameters being changed, their rate of change, and the masking properties of the human ear. Reference [2] offers practical strategies for updating filter parameters to minimize audible effects. The useful aspects of these strategies are pointed out when relevant.

This paper investigates real-time control of a typical parametric filter section, along with the filter topology used to implement it in the digital domain. Computational requirements for digital filter coefficients are shown, and a description is offered of a system implementation which allows listening tests to be performed. The main goal of this paper is to determine the audibility of artifacts that occur when filter parameters are updated, and the conditions for reducing or eliminating them.

2 PARAMETRIC FILTER

The parametric filter is so called since its response is specified by three parameters: center frequency, bandwidth (or Q), and gain (peak boost or cut). The transfer function is second order, and is often referred to as a biquadratic. The analog domain transfer functions for boost ($H_B(s)$) and cut ($H_C(s)$) are given by equations (1) and (2) respectively. A plot of a typical parametric filter response is shown in Figure 2-1.

Parametric equalizers are constructed by cascading several parametric filter sections. Usually, the center frequency, Q, and desired boost or cut of each section can be adjusted.

$$H_B(s) = \frac{s^2 + \frac{K\Omega_0}{Q}s + \Omega_0^2}{s^2 + \frac{\Omega_0}{Q}s + \Omega_0^2} \tag{1}$$

$$H_C(s) = \frac{s^2 + \frac{\Omega_0}{Q}s + \Omega_0^2}{s^2 + \frac{K\Omega_0}{Q}s + \Omega_0^2} \tag{2}$$

where,

$\Omega_0 = 2\pi F_0$, where F_0 is the center frequency in Hz;

$Q = \frac{F_0}{BW}$, where BW is the 3 dB bandwidth in Hz; and

$K = 10^{\frac{G}{20}}$, where G is the desired boost or cut in dB.

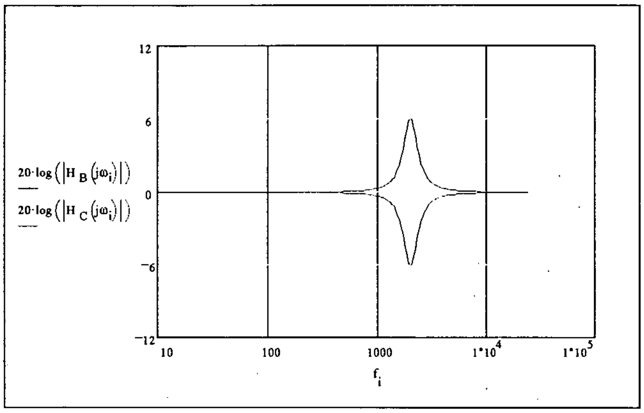

Figure 2-1. Parametric filter response for F_0 = 2000 Hz, Q = 4, and boost/cut = 6 dB

3 FILTER TOPOLOGY

In this investigation, the parametric filter described above will be implemented in the digital domain as a recursive filter (i.e. Infinite Impulse Response or IIR filter). Since real DSPs offer only finite precision arithmetic, the performance of the filter depends on its implementation, just as an analog filter's performance depends on its circuit topology. There are many DSP filter implementations, each with different performance and complexity.

Generally, there is a tradeoff between the advantages of given structure and its computational complexity. These tradeoffs involve stability, coefficient sensitivity, scaling issues, limit cycles and truncation noise. In a fixed precision digital filter, truncation results in nonlinear behavior. Truncation noise becomes significant at low signal levels, and, since it is highly correlated with the signal, it is spectrally not white. In the direct form I structure, truncation noise is amplified by the poles of the filter and becomes worse at low pole frequencies.

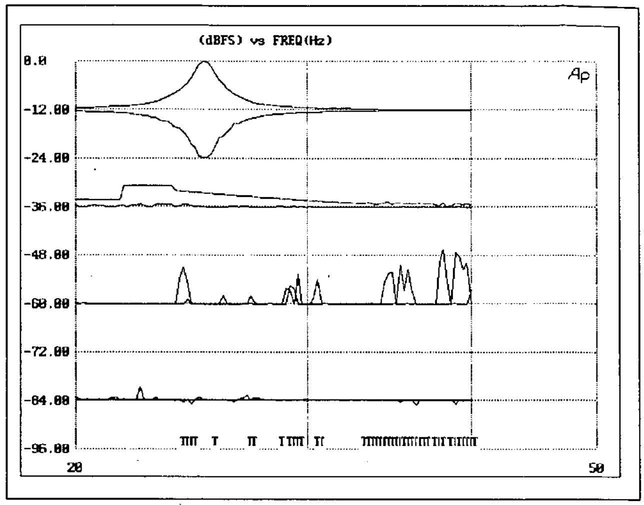

Figure 3-1 Parametric Filter Response at Various Input Levels, Single Precision Direct Form I

3.1 Direct Form I Topology

Figures 3-1 illustrates the effect of the truncation noise on filter performance for the direct form I structure. The curve shows the magnitude response vs. frequency of a parametric filter as the input level is decreased. This parametric filter has a center frequency of 25 Hz, a bandwidth of 0.1 octaves, and a gain of +/- 12 dB (boost and cut). It was implemented on a 24 bit DSP with single-precision arithmetic. Note how quickly the performance degrades. Even at -36 dB, the filter response is completely obscured by truncation noise. Obviously, the direct form I topology by itself is unusable for high fidelity audio applications. Figure 3-2 shows the same filter using double-precision, with performance as expected. However, a full double precision implementation has a much higher computational requirement.

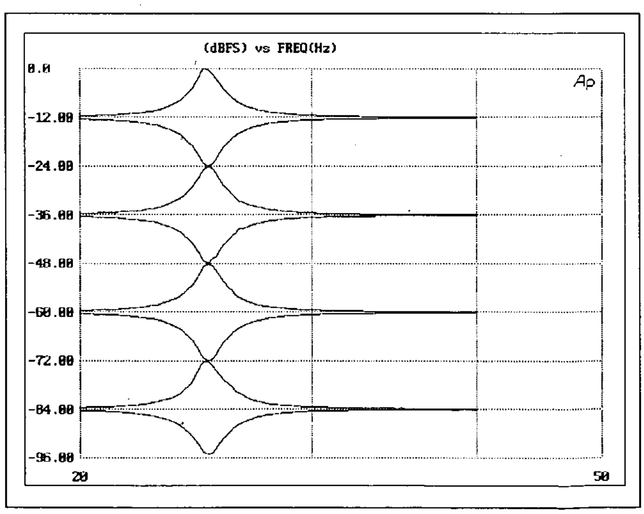

Figure 3-2. Parametric Filter Response at Various Input Levels, Double Precision Direct Form I

3.2 Direct Form I Topology With Error Spectrum Shaping

The performance limitations of the standard direct form I implementation can be overcome by augmenting it with error spectrum shaping (ESS). ESS is a technique which introduces zeros into the filter *noise* transfer function without affecting the *signal* transfer function. This significantly reduces the error amplification. Optimal second order ESS essentially amounts to using double precision arithmetic for the recursive part of the filter. Reference [6] provides a very thorough discussion on the theory and application of ESS. Reference [3] provides an excellent discussion of how even first order ESS significantly improves the truncation noise performance of the direct form I topology.

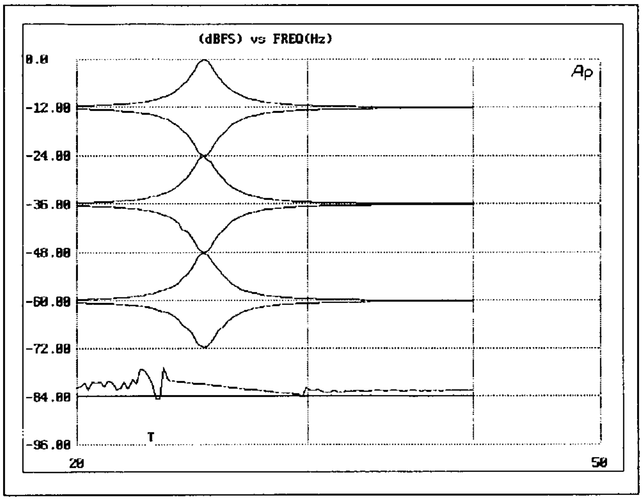

Figure 3-3. Parametric Filter Response at Various Input Levels, Single Precision Direct Form I with First Order ESS

Figure 3-3 shows the filter implemented with single-precision and first order (zero at dc) ESS. The response is significantly improved, but this filter still does not have enough dynamic range for professional audio applications.

Figure 3-4 shows the filter implemented with single-precision and second order (pole cancellation) ESS. This response is identical to that of the double-precision implementation. Figure 3-5 shows that second order ESS preserves the desired response down to the noise floor of the system. The second order ESS implementation requires less computation than double precision. Therefore, the filter topology used in this report is the direct form I using second order ESS. This topology is fairly well accepted for high dynamic range digital audio.

It should be noted that these filters were implemented on a system with good input SNR. On systems with a higher noise floor, the extra input noise reduces truncation noise

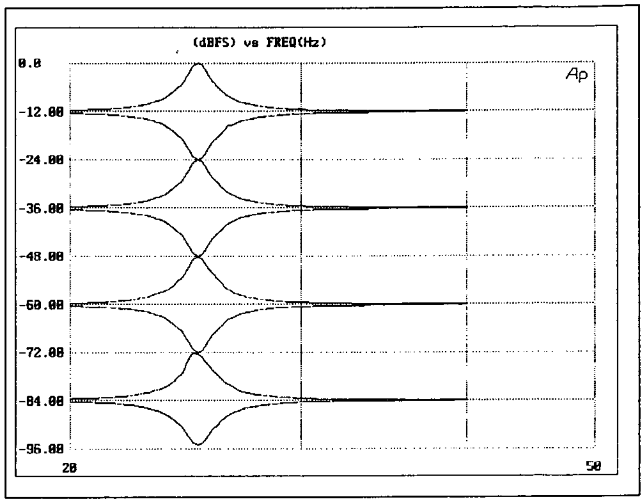

Figure 3-4. Parametric Filter Response at Various Input Levels, Single Precision Direct Form I with Second Order ESS

by linearizing the quantization. Of course, the higher noise floor reduces the system dynamic range.

3.3 Other Issues

Only the direct form I topology with second order ESS is investigated in this paper. Future work should consider other filter topologies. References [3,4,5] provide useful information on the time-invariant performance of various filter structures. For the case of time-varying digital filters, the issues of time-varying stability and complexity of computing coefficients in real-time must be considered. A quantitative discussion of the time-varying stability issue is beyond the scope of this paper. Instead, a qualitative assessment was made based on listening tests. Future work should also investigate the performance of different topologies under time-varying conditions.

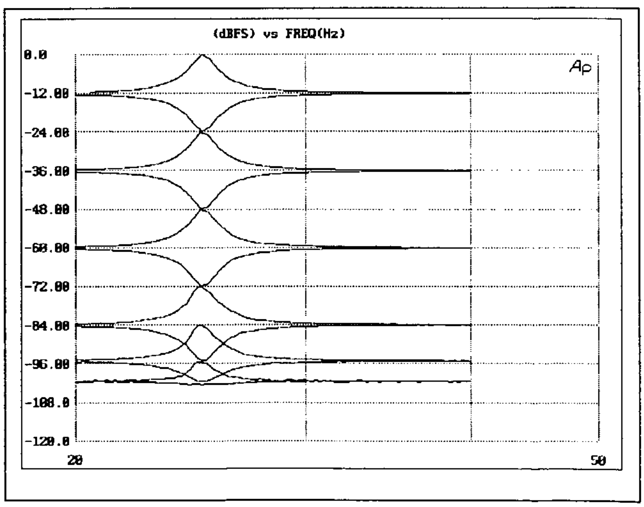

Figure 3-4. Parametric Filter Response at Input Levels to the Noise Floor, Single Precision Direct Form I with Second Order ESS

4 COEFFICIENT CALCULATIONS

The transfer function for the direct form I topology is given by (3). It is implemented by the second order difference equation given by (4). The additional terms used to implement the second order ESS are not shown for clarity. Since the ESS implemented here uses the same coefficients as the denominator of (3), no additional computation is required to compute the filter coefficients.

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{3}$$

$$\begin{aligned} y[n] &= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \\ &\quad - a_1 y[n-1] - a_2 y[n-2] \end{aligned} \tag{4}$$

For the parametric filter, the digital filter coefficients are determined by F_o, Q, G, and the sample rate F_s. The equations for the coefficients were determined by the approach given in [7], which is also followed by [1,5]. It should be noted that these equations result in different bandwidth for boost and cut. If desired, symmetrical boost and cut response may be accomplished by scaling the bandwidth by 1/K when K is less than one.

It is useful to divide the calculation of the coefficients up into two steps, first calculating a set of intermediate values followed by the calculation of the filter coefficients from those intermediate values. The intermediate calculations are given in Table 4-1. The calculations for the filter coefficients are given in Table 4-2.

$b = -\cos(2\pi F_0/Fs)$
$a = \dfrac{1-\tan(\pi B/Fs)}{1+\tan(\pi B/Fs)} \quad where\, B = F_0/Q$
$K = 10^{G/20} \quad where\, G\, is\, gain\, at\, F_0\, in\, dB$

Table 4-1. Intermediate Calculations for Filter Coefficients

$b_0 = (1+a+K-Ka)\frac{1}{2}$
$b_1 = (b+ba)$
$b_2 = (1+a-K+Ka)\frac{1}{2}$
$a_1 = b1$
$a_2 = a$

Table 4-2. Filter Coefficient Calculations

Note that the calculations in Table 4-1 involve a fair amount of computation. However, assuming F_s is fixed, **a** depends only on the center frequency, **b** depends only on the bandwidth, and **K** depends only on the gain. In many systems it is reasonable to assume that these parameters would be precomputed and stored in memory. For example, consider a parametric filter with 60 possible center frequencies (1/6 octave), 10 possible Q values, and 60 possible gain values (+/-15 dB in 0.5 dB steps). This would require only 480 memory locations for each band of parametric EQ. Note that because the calculations in Table 4-2 involve only multiplies and adds, they are suited to being computed in real-time by a DSP.

Note that for this topology there is not a one-to-one mapping between the filter parameters and the coefficients. Topologies exist for which there is a one-to-one mapping [7]. Reference [5] further discusses the properties of this topology which indicates that it may be desirable in a time-varying application. This work also addresses the computational aspects, which are higher than those of the structure employed here.

5 PARAMETER VARIATION IN REAL-TIME

In order to provide real-time control over the settings of a parametric equalizer, the coefficients must be changed successively from their present state, through a series of intermediate states, to their desired state. It is obvious that audible discontinuities will be produced if a parameter such as center frequency is abruptly changed. At some small level of change per step, the audible effects of step-wise parameter updates should disappear, and the filter should sound subjectively the same as a swept analog filter. However, identifying the threshold at which such step-wise changes become discernible as different from continuous analog control is of importance to the DSP engineer.

The transition from one set of filter parameters to another is accomplished as indicated in Figure 5-1. Here, ΔP is the total amount of parameter variation over the time

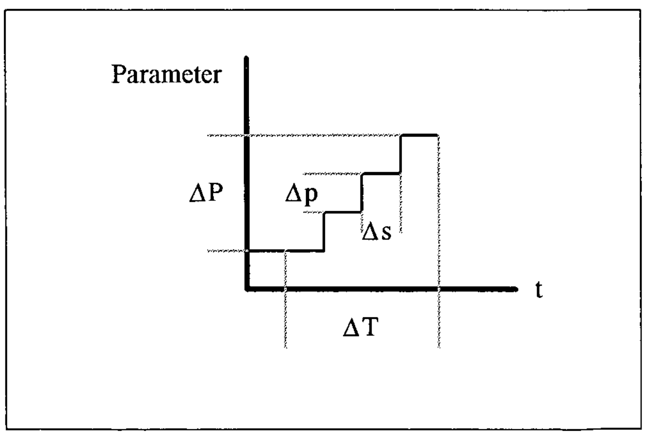

Figure 5-1 Transition of Filter Parameters

interval ΔT, where ΔP corresponds to either ΔF in Hz, ΔQ in units, or ΔG in dB. The quantity Δs has units of seconds/state, and is the amount of time between successive filter updates. The quantity Δp is the amount of parameter variation per filter update.

These quantities are related as shown in equations (5-7). Typically, the amount of parameter variation ΔP and the transition time interval ΔT will be given. What needs to be determined is the rate of filter updates, given by $1/\Delta s$, and thus the granularity of parameter variation, Δp. The ratio $\Delta P/\Delta p$ represents the number of filter states required during the transition.

$$\Delta F/\Delta T = \Delta f/\Delta s \quad \text{Hz/s} \tag{5}$$

$$\Delta Q/\Delta T = \Delta q/\Delta s \quad \text{units/s} \tag{6}$$

$$\Delta G/\Delta T = \Delta g/\Delta s \quad \text{dB/s} \tag{7}$$

6 SYSTEM IMPLEMENTATION

The system used to evaluate real-time control of the parametric filter is shown in Figure 6-1. The analog output from a CD player was used as a signal source. The signal was converted to digital form by the A/D and provided to a DSP on a PC plug in board at a baseband sample rate of 48 kHz. The DSP provided output samples at this same rate to the D/A converter. The resulting analog signal was then amplified and auditioned over a loudspeaker.

The DSP board used was based on the Motorola 56001, a 24 bit fixed point processor. The PC used here was a 486DX2 (66 MHz) machine. The software running in the DSP implemented the parametric filter and accepted filter coefficient updates in real-time. The DSP automatically synchronized filter updates to the sample rate. Software running on the PC allowed the user to specify the initial and target set of filter parameters (i.e. ΔF, ΔQ, ΔG), the parameter step sizes (Δf, Δq, Δg), and the update period (Δs). The software was flexible enough to allow all parameters to be varied simultaneously, or one at a time.

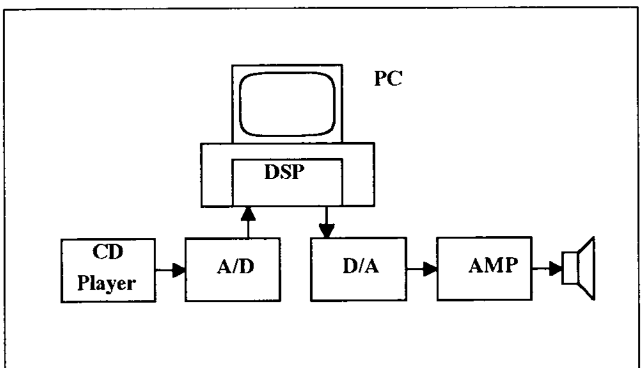

Figure 6-1. Test System Block Diagram

In the PC software, the time between filter updates was controlled accurately by reprogramming the system timer interrupt, which allowed timing resolution to be much better than the sample period. Other interrupts were disabled during the update. At each update interrupt, the PC computed the new filter parameters, computed the corresponding filter coefficients, and downloaded them to the DSP board. On the PC used here, this process took about 400 μs. The filter coefficient calculation itself took about 150 μs. The time between filter updates could be programmed from 500 μs up to 1 second.

Despite the expectation that instability might arise, no sign of instability was encountered during parameter updates using the direct form I topology with second order ESS in the tests performed here.

7 TESTS AND RESULTS

The experimental objective was to determine the real-time requirement for parametric filter updates based on subjective listening tests. Specifically, for a given change in filter parameter over a given period of time, what was the required update rate for subjectively smooth change? By varying one filter parameter at a time, i.e. F, Q, or G, it was possible to make an informal assessment of the audibility of discontinuities and distortions caused by the parameter variation.

Two types of signal sources were used as test signals: sine waves and music. Two types of music sources were used during testing, solo piano and solo flute. The effects of parameter variation were most audible on sine waves. When music with broadband character was used, the effects were much less audible. Solo piano and solo flute seemed to be a reasonable compromise between pure tone sine waves and highly broadband music.

As mentioned above, the listening tests reported herein were informal. No attempt was made to ensure blind listening conditions. The subjective opinions herein represent only the author's reaction to the test stimuli. Nonetheless, casual experimentation with others in the author's lab indicated general agreement with the subjective assessment presented herein. More rigorous, listening tests with multiple subjects would provide an interesting avenue for further study.

Δs (mS)	Δg (dB)	ΔT (S)	1 kHz sine wave (F = 1 kHz)	Solo Piano (F = 1 kHz)	Solo Flute (F = 400 Hz)
100	1	1	Audible discrete steps	Inaudible effect	Audible discrete steps
100	0.1	10	Slightly audible discrete steps	Inaudible effect	Slightly audible discrete steps
10	1	0.1	Audible "zipper" noise	Inaudible effect	Audible "zipper" noise
10	0.1	1	Audible "zipper" noise	Inaudible effect	Inaudible effect
1	1	0.01	Inaudible effect	Inaudible effect	Inaudible effect
1	0.1	0.1	Inaudible effect	Inaudible effect	Inaudible effect
1	0.01	1	Inaudible effect	Inaudible effect	Inaudible effect

Table 7.1 Results of Gain Variation (ΔG=10 dB, Q=2)

7.1 Gain Variation

For the gain variation tests, the frequency and Q were held constant. Each filter update changed only the gain. Gain changes were made in equal dB steps. Various values of frequency and Q were used.

Table 7.1 shows representative results of the listening tests that were performed. In these tests, gain G was varied from 0 to 10 dB of boost (i.e. ΔG = 10 dB), Q = 2, and F = 1 kHz (F = 400 Hz for flute). The results indicate that a fast update rate is required (Δ s= 1 ms) before effects from gain variation become inaudible. At this rate even large gain steps (Δg = 1 dB) do not produce objectionable effects since the gain change happens very fast. If slower gain changes are required, a smaller gain step (Δg) should be used. Testing was repeated with values of Q up to 14 with no change in the audible effects. Testing was repeated with various center frequency values. The audibility of the discrete steps and "zipper" effects decreased with decreasing frequency.

Δs (mS)	F step (octave)	1 kHz sine wave	Solo Piano	Solo Flute
100	1/3	Audible discrete steps	Inaudible effect	Audible discrete steps
100	1/10	Audible discrete steps	Inaudible effect	Audible discrete step.
100	1/30	Audible discrete steps	Inaudible effect	Slightly audible discrete steps
100	1/60	Audible discrete steps	Inaudible effect	Slightly audible discrete steps
10	1/3	Audible "zipper" noise	Slightly audible "zipper" noise	Slightly audible "zipper" noise
10	1/10	Audible "zipper" noise	Slightly audible "zipper" noise	Slightly audible "zipper" noise
10	1/30	Audible "zipper" noise	Inaudible effect	Slightly audible "zipper" noise
10	1/60	Audible "zipper" noise	Inaudible effect	Slightly audible "zipper" noise
1	1/3	Audible effect due to fast sweep	Inaudible effect	Audible effect due to fast sweep
1	1/10	Audible effect due to fast sweep	Inaudible effect	Audible effect due to fast sweep
1	1/30	Slightly audible effect	Inaudible effect	Inaudible effect
1	1/60	Slightly audible effect	Inaudible effect	Inaudible effect

Table 7.2 Results of Frequency Variation (G=6 dB, Q=2, F swept from 100 Hz to 1 kHz)

7.2 Frequency Variation

For the frequency variation tests, the gain and Q were held constant. Each filter update changed only the center frequency. Frequency changes were made in logarithmic steps, specified in fractions of an octave, therefore the Δf parameter is not a constant, which must be taken into account when using (5). Various values of gain and Q were used.

Table 7.2 shows representative results for G = 6 dB, Q = 2, and F varying from 100 Hz to 1 kHz. Most of the tests resulted in audible zipper noise, even at very small frequency steps. A fast update rate is required (Δs = 1 ms) before the zipper effects become inaudible. The audibility of the discrete steps and "zipper" effects decreased with decreasing frequency.

Testing was repeated with values of Q up to 14 with only very slight changes in the character of the audible effects. Testing was repeated with various gain values. When G = 0 dB, no audible effects were produced during frequency variation. While this may at first seem obvious, changing the center frequency invariably changes the digital filter coefficients. This result suggests that the strategy used in [2] is worthwhile if the gain may be set to 0 dB before changing the frequency (unfortunately for this method, such is not always the case).

7.3 Q Variation

For the Q variation tests, the frequency and gain were held constant. Each filter update changed only the Q. The Q was changed in linear steps. Various values of frequency and gain were used. Table 7.3 shows representative results. Here, Q was varied from 2 to 12 ($\Delta\Theta$ = 10), G = 6 dB, and F = 1 kHz (F = 400 Hz for flute). Tests for sine wave inputs used three different frequencies as shown in the table. For sine wave inputs, the audibility of artifacts increased with increasing frequency. With solo piano, no audible effects were detected. For solo flute, effects were detected even at Δs = 1 ms with ΔQ = 1. At Δs = 1 ms with ΔQ = 0.1, no effects were detected.

8 CONCLUSION

Digital domain time-varying filters can be used to approximate their analog counterparts. Not surprisingly, in general, the more closely a parameter update approached a continuous function, the less audible was the effect of the parameter variation. These results indicate that frequency variation in parametric equalizers produces the greatest audible effect. A reasonably fast update rate (around 1 ms) was required before effects became inaudible on solo program material. This rate is demanding in terms of the computational power required to compute updated filter parameters in real-time.

9 ACKNOWLEDGMENTS

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Δs (mS)	ΔQ (units)	Sine waves (125 Hz, 1 kHz, 4 kHz)	Solo Piano	Solo Flute (F = 400Hz)
100	1	Inaudible effect at 125 Hz and 1 kHz Audible discrete steps at 4 kHz	Inaudible effect	Slightly audible discrete steps
100	0.1	Inaudible effect at 125 Hz and 1 kHz Audible discrete steps at 4 kHz	Inaudible effect	Inaudible effect (long sweep ΔT=10s)
10	1	Inaudible effect at 125 Hz and 1 kHz Audible zipper effect at 4 kHz	Inaudible effect	Slightly audible zipper effect
10	0.1	Inaudible effect at 125 Hz and 1 kHz Audible zipper effect at 4 kHz	Inaudible effect	Inaudible effect
1	1	Inaudible effect at 125 Hz and 1 kHz Audible effect at 4 kHz due to fast sweep	Inaudible effect	Slightly audible zipper effect
1	0.1	Inaudible effect at 125 Hz and 1 kHz Audible zipper effect at 4 kHz	Inaudible effect	Inaudible effect
1	0.01	Inaudible effect at 125 Hz and 1 kHz Slightly audible zipper effect at 4 kHz	Inaudible effect	Inaudible effect

Table 7.3. Results of Q Variation (G=6 dB, Q swept from 2 to 12)

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